ELECTIVE PAPER

PHYSICS AT LHC AND BEYOND

UNIT I

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Knowledge About

- Lagrangian & Hamiltonian formalism
- Schrodinger, Klein-Gordon and Dirac equations
- Mechanics to field theory
- Elementary particles
- Standard Model
- Higgs field & Mass generation
- Symmetry

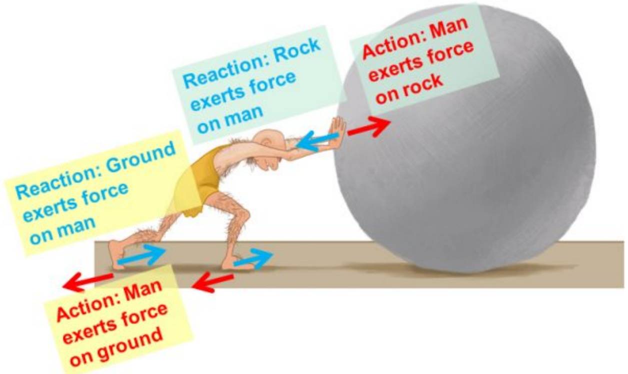
Introduction

- Let us begin by considering the simplest possible system in classical mechanics, a single point particle of mass m in one dimension, whose coordinate and velocity are functions of time, x(t) and υ = dx(t)/dt, respectively. Let the particle be exposed to a time-independent potential V (x). It's motion is then governed by Newton's law. F = ma
- $$m\frac{d^2x}{dt^2} = -\frac{\partial V}{\partial x} = F(x)$$

- By knowing the initial conditions and the equations of motion, we also know the evolution of the particle at all times (provided we can solve the equations of motion)

Lagrangian formalism in classical mechanics

- The equation of motion in the form of Newton's law was originally formulated as an equality of two forces, based on the physical principle action = reaction, i.e. the external force is balanced by the particle's inertia.



- The Lagrangian formalism is formal, rather than physical. It is an immensely useful tool allowing to treat all kinds of physical systems by the same method.

- Lagrange function $$L(x, \dot{x}) = T - V = \frac{1}{2} m \dot{x}^2 - V(x)$$

- Euler-Lagrange equation

$$\boxed{\frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = 0}$$

Hamiltonian formalism

$$H(x,p) = \ \tfrac{1}{2}m\dot{x}^2 + V(x) = T + V.$$

$$\frac{\partial H}{\partial x} = -\dot{p}, \quad \frac{\partial H}{\partial p} = \dot{x}$$

These are two equations of first order, while the Euler-Lagrange equation was a single equation of second order.

Schrodinger equation (Non-Relativistic)

$$H\psi = E\psi$$

Take as the starting point non-relativistic energy

$$E = T + V = \frac{\vec{p}^2}{2m} + V$$

(take V=0 for simplicity)

Kinetic energy

$$E = p^2/2m$$

Physical quantities are represented by operators

$$E \leftrightarrow i\hbar\frac{\partial}{\partial t}, \qquad \mathbf{p} \leftrightarrow -i\hbar\boldsymbol{\nabla}$$

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi$$

The SE is first order in the time derivatives and second order in spatial derivatives – and is manifestly not Lorentz invariant.

plane wave solutions: $\psi = Ne^{i(p.r - Et)}$

Wave function (ψ) is a probability amplitude, whose modulus squared is the probability of finding the particle at a particular position (or with a particular momentum).

The probability density $\rho = |\psi|^2$, which is associated with a probability current density

$$j = -i(\psi^* \nabla \psi - \psi \nabla \psi^*)$$

$$\rho = |N|^2 \quad \text{and} \quad j = |N|^2 \frac{p}{m} = |N|^2 v$$

For $|N|^2$ particles per unit volume moving at velocity v, have $|N|^2 v$ passing through a unit area per unit time (particle flux).

Probability conservation equation make up together

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{J} = 0$$

Schrodinger wanted a relativistic wave equation describing the electron

Klein-Gordon (relativistic)

- The relativistic relation between energy and momentum

$$E^2 = p^2 + m^2$$

c = 1 for simplicity

$$E = \pm\sqrt{p^2 + m^2}$$

- Relativistic version of the equation $E \leftrightarrow i\hbar\frac{\partial}{\partial t}, \quad \mathbf{p} \leftrightarrow -i\hbar\nabla$

$$-\frac{\partial^2}{\partial t^2}\phi = -\nabla^2\phi + m^2\phi$$

$$(\partial_t^2 + m^2)\,\phi = 0$$

which is called "d'Alembertian

$\partial_t^2 \equiv \square$

- The solutions to this equation are $\phi = Ae^{i(p.r - Et)}$

A an arbitrary complex constant

(It is traditional to use ϕ rather than ψ in this context. ϕ is a solution to the Klein-Gordon equation it cannot be interpreted as a wave function as Schrodinger discovered

The KG equation also has a conserved density and a 3-vector current, which are

$$\rho = -i/2\,(\phi^* \partial_t \phi - \phi\, \partial_t \phi^*)\,, \qquad j = -i/2\,(\phi^* \nabla \phi - \phi \nabla \phi^*)$$

However, this density cannot be interpreted as a probability density, as it is not positive denite. This was the reason that Schrodinger chose the non-relativistic form for his equation. This was reasonable for his purposes, but the Klein-Gordon equation makes a comeback later, when we shall see that can be interpreted as a charge density, which is allowed to take both positive and negative values.

$$E = \pm\sqrt{p^2 + m^2}$$

$$\rho = 2E|A|^2 \text{ and } j = |A|^2 p$$

Historically, it was thought that there were two main problems

Negative energy solutions

The negative particle densities associated with these solutions

- The Klein-Gordon (KG) equation (called also Schroedinger's relativistic wave equation)

  Schrodinger used Klein-Gordon equation with potential to tackle the hydrogen atom problem. However, he found out that the obtained result didn't agree with the experiments.

- It was later found out that Klein-Gordon equation did not yield the right answer for the hydrogen atom problem, as the fact that an electron has spin 1/2 was not considered in solving the problem.

- Klein-Gordon field describes particles that have spin 0. Such fields necessarily don't have any indices, as the wave function of spin 0 particles do not transform under rotation; they are scalar fields.

Schrödinger eq $$-\frac{1}{2m}\vec{\nabla}^2\psi = i\frac{\partial\psi}{\partial t}$$

1st order in $\partial/\partial t$

2nd order in $\partial/\partial x, \partial/\partial y, \partial/\partial z$

Klein-Gordon eq $(\partial_t^2 + m^2)\,\phi = 0$

2nd order throughout

Dirac looked for an alternative which was 1st order throughout

$$\hat{H}\psi = (\vec{\alpha}.\vec{p} + \beta m)\psi = i\frac{\partial\psi}{\partial t}$$

$$
\begin{aligned}
-\frac{\partial^2\psi}{\partial t^2} \quad = \quad & -\alpha_x^2\frac{\partial^2\psi}{\partial x^2} - \alpha_y^2\frac{\partial^2\psi}{\partial y^2} - \alpha_z^2\frac{\partial^2\psi}{\partial z^2} + \beta^2 m^2\psi \\
& -(\alpha_x\alpha_y + \alpha_y\alpha_x)\frac{\partial^2\psi}{\partial x\partial y} - (\alpha_y\alpha_z + \alpha_z\alpha_y)\frac{\partial^2\psi}{\partial y\partial z} - (\alpha_z\alpha_x + \alpha_x\alpha_z)\frac{\partial^2\psi}{\partial z\partial x} \\
& -(\alpha_x\beta + \beta\alpha_x)m\frac{\partial\psi}{\partial x} - (\alpha_y\beta + \beta\alpha_y)m\frac{\partial\psi}{\partial y} - (\alpha_z\beta + \beta\alpha_z)m\frac{\partial\psi}{\partial z}
\end{aligned}
$$

For this to be a reasonable formulation of relativistic QM, a free particle must also obey $E^2 = \vec{p}^2 + m^2$, i.e. it must satisfy the Klein-Gordon equation:

$$
-\frac{\partial^2\psi}{\partial t^2} = -\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial z^2} + m^2\psi
$$

Hence for the Dirac Equation to be consistent with the KG equation require:

$$
\begin{aligned}
\alpha_x^2 = \alpha_y^2 = \alpha_z^2 = \beta^2 &= 1 \\
\alpha_j\beta + \beta\alpha_j &= 0 \\
\alpha_j\alpha_k + \alpha_k\alpha_j &= 0 \qquad (j \neq k)
\end{aligned}
$$

The wave-function must be a four-component Dirac Spinor

$$
\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}
$$

- Unlike the KG equation, the Dirac equation has probability densities which are always positive.
- In addition, the solutions to the Dirac equation are the four component Dirac Spinors. A great success of the Dirac equation is that these components naturally give rise to the property of intrinsic spin.
- It can be shown that Dirac spinors represent spin-half particles with an intrinsic magnetic moment of

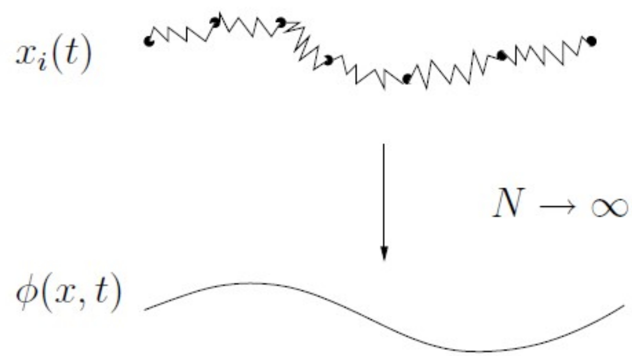
Interpretation of –ive Energy Solutions

Dirac Interpretation: the vacuum corresponds to all –ve energy states being full with the Pauli exclusion principle preventing electrons falling into -ive energy states. Holes in the –ive energy states correspond to +ve energy anti-particles with opposite charge.

Anti-particle solutions exist ! But the picture of the vacuum corresponding to the state where all -ive energy states are occupied is rather unsatisfactory, what about bosons (no exclusion principle)

Feynman : Interpret a negative energy solution as a negative energy particle which propagates backwards in time or equivalently a positive energy anti-particle which propagates forwards in time

From N-point mechanics to field theory



The displacement of the string at some particular point x along its length is given by a field coordinate $\phi(x, t)$

$$
\begin{array}{rcl}
\text{Classical Mechanics:} & & \text{Classical Field Theory} \\
x(t) & \longrightarrow & \phi(x,t) \\
\dot{x}(t) & \longrightarrow & \dot{\phi}(x,t) \\
i & \longrightarrow & x \\
L(x,\dot{x}) & \longrightarrow & \mathcal{L}[\phi,\dot{\phi}]
\end{array}
$$

$L[\phi, \dot{\phi}]$ depends on the functions $\phi(x,t), \dot{\phi}(x,t)$ at every space-time point, but not on the coordinates directly.

Lagrangian of the electromagnetic field

The following Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

gives rise to the Maxwell equations as classical equations of motion

Quantising the classical Lagrangian of the electromagnetic field failed

Consider a modified Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{2} \left(\partial_\mu A^\mu\right)^2$$

Lagrangian for the Dirac field

$$(i\gamma^\mu \partial_\mu - m)\,\psi = 0$$

$$\not{a} \equiv \gamma^\mu a_\mu, \qquad \not{\partial} \equiv \gamma^\mu \partial_\mu$$

$$(i\not{\partial} - mI_{4\times 4})\,\psi = 0$$

To make the transition to field theory, now ψ is interpreted as a field, rather than a quantum state

$$\mathcal{L}_0^{1/2} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$$

The $\frac{1}{2}$ superscript on $\mathcal{L}$ signifies that we're dealing with spin-$\frac{1}{2}$ particles (and is *not* a square root!), while the subscript 0 indicates that we're dealing with free particles. [It's worth remembering at this point that this is a matrix equation; $\bar{\psi}$ is a 4-d row vector, ψ is a 4-d column vector, γ^μ is a 4×4 matrix and m is multiplied by the 4×4 identity matrix. After multiplying all these matrices together, though, the Lagrangian density is a scalar.]

Elementary particles

- An elementary particle is often characterized by its fixed mass, spin/angular momentum, intrinsic parity, charge, electric and magnetic moments and other quantum numbers like baryon and lepton number, strangeness, charm, color, weak isospin, weak hypercharge etc. Most of these attributes are associated with space-time or internal symmetries.

- No sub-structure, No excited state.

- Wigner's theorem (1939) gives a way of thinking about an elementary particle: one whose quantum states (labeled by momentum and spin projection) carry an irreducible unitary representation of the Poincare group of space-time symmetries. The relevant irreducible representations of the Poincare group are labeled by mass and spin. The group of space-time symmetries may be extended to include internal symmetries like electromagnetic gauge symmetry, leading to additional quantum numbers.

Standard Model

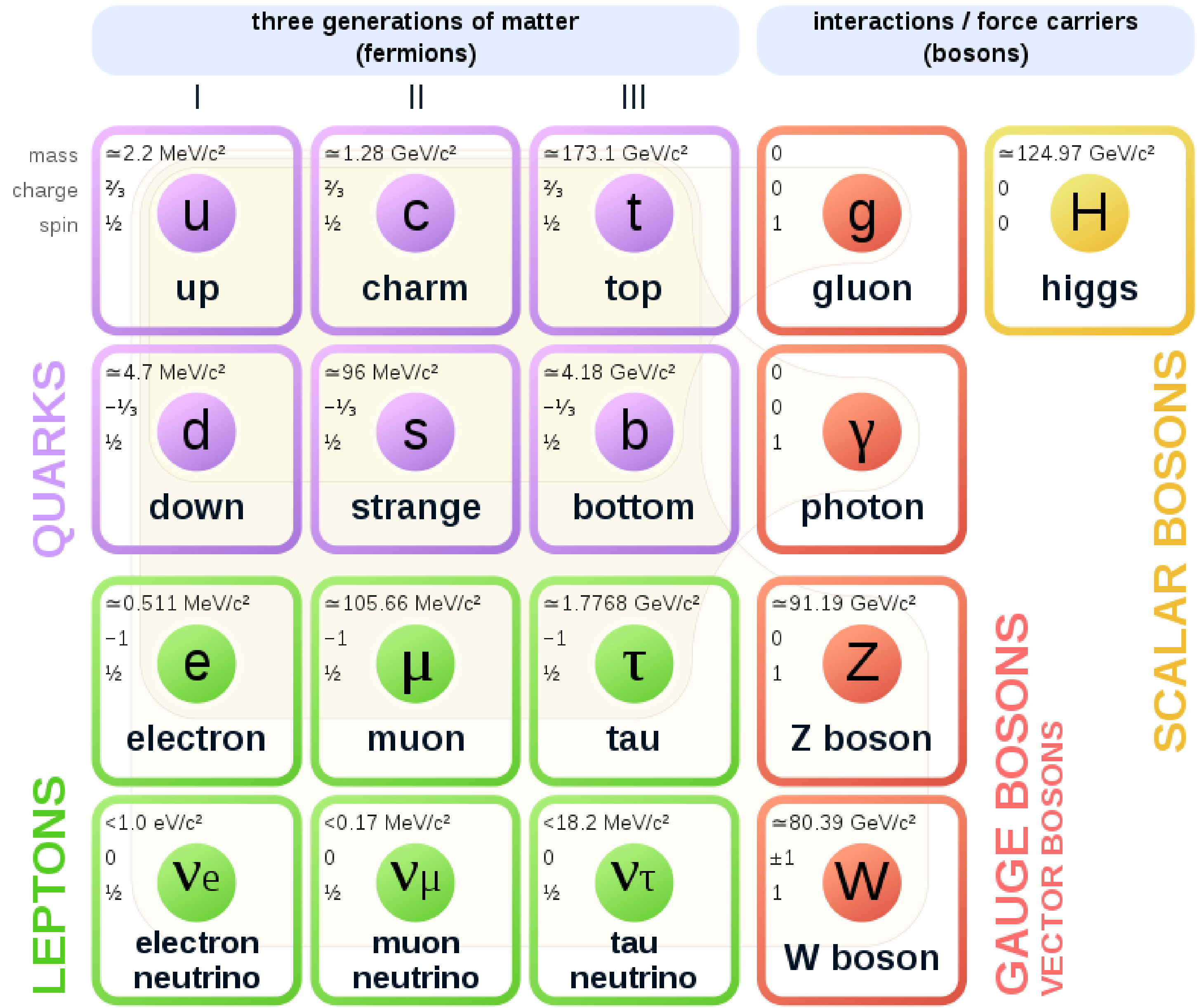
- The standard model (**SM**) of particle physics is theory of elementary particles. It is a remarkably successful and elegant relativistic quantum field theory based on the ‘gauge principle’ and ‘renormalizability’.

- The particles are roughly divided into matter particles (fermionic spin-half quarks and leptons) and force carriers (spin one gauge bosons) and a spin zero Higgs boson.

- To each elementary particle of the SM, there is associated a quantum field whose elementary excitation (produced by a creation operator acting on the vacuum) is the particle.

- The **SM** is the theory describing three of the four known fundamental forces, the electromagnetic, weak, and strong interactions, and not including the gravitational force.

Standard Model of Elementary Particles



- The Standard Model (SM) consists of the electro-weak theory and the theory for the strong interactions known as QCD. According to this model, all matter consists of point-like particles which are either quarks or leptons (called fermions), whereas the forces between these particles are mediated by the exchange of intermediate bosons: the gluon, the photon, the Z and the W + /−.

- There are 3 families or ”generations”, but only the particles of the first family are believed to exist as stable particles in nature (the single quarks are not expected to exist as free particles but assumed to be bound within the protons). The leptons and the quarks of the second and third family are only present in extreme energy situations available at HEP accelerators or in cosmic rays. These energies were present, according to cosmological theory, right after the Big Bang.

- Each lepton and quark has an anti-particle partner and all quarks have 3 different ”colors”.

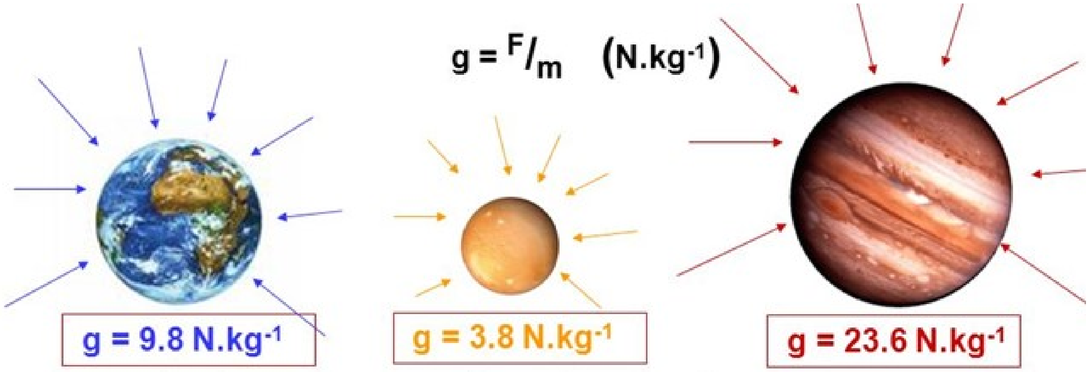
- There are 8 types of gluons (also associated with ”color”),

- The Standard Model operates with 48 matter particles and 12 force mediators. All other particles are assumed to be combinations of these constituents.

- Goldstone bosons and the Higgs bosons are generated by symmetry breaking.

What is Mass?

Each kilogram of mass experiences of force



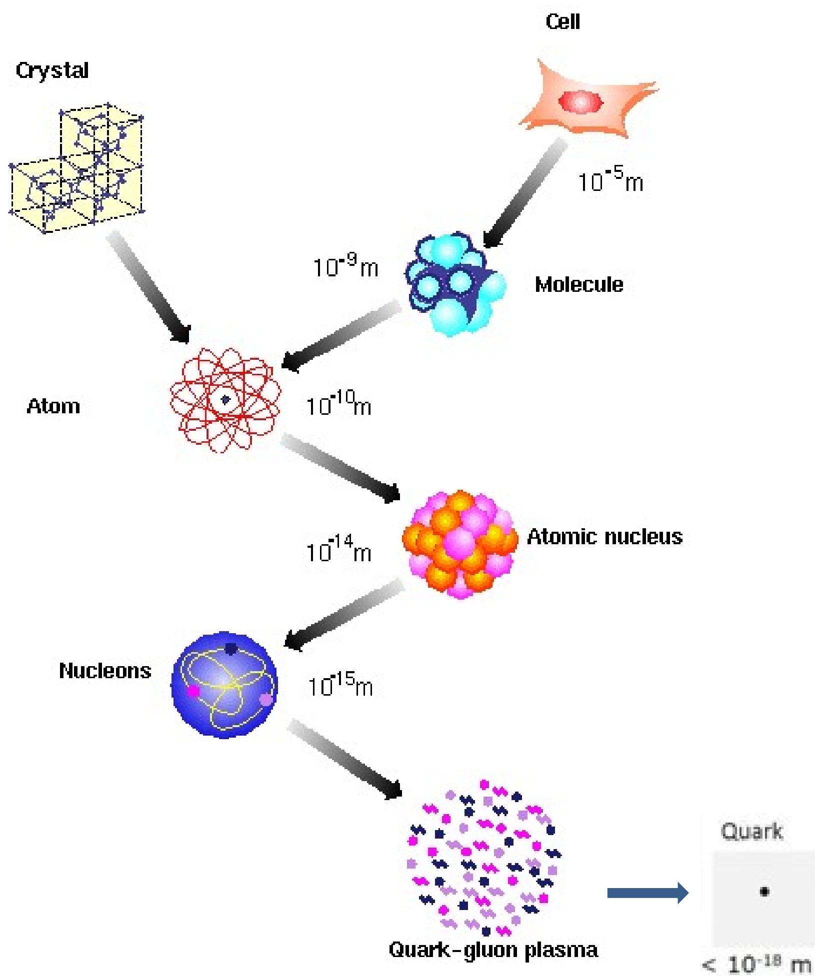
Mass and Energy are equivalent

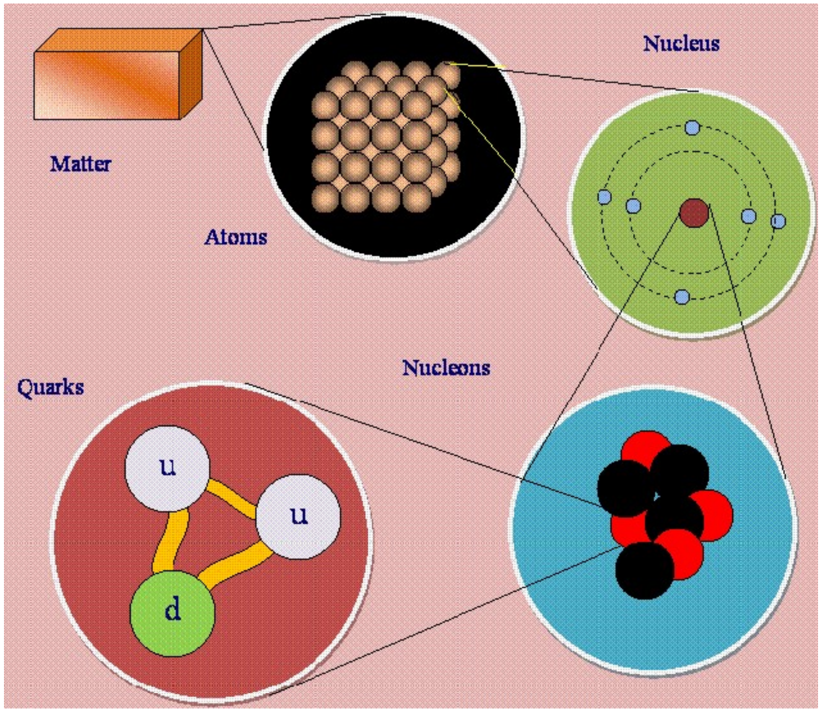
$$E = mc^2$$ in Special Relativity

Mass is important even without Gravity (e.g. in vacuum)

What gives us mass?

Building Blocks of Matter





$$m_{\rm body} \approx N_p\, m_{\rm proton} + \underbrace{N_n\, m_{\rm neutron}}_{}$$

building blocks for mass of body

$$m_{\rm proton} \approx m_{\rm neutron}$$

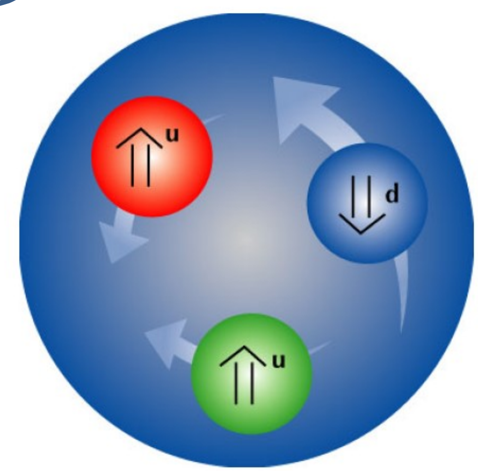
$$\boxed{m_{nucleus} < N_p\, m_{\rm proton} + N_n\, m_{\rm neutron}}$$

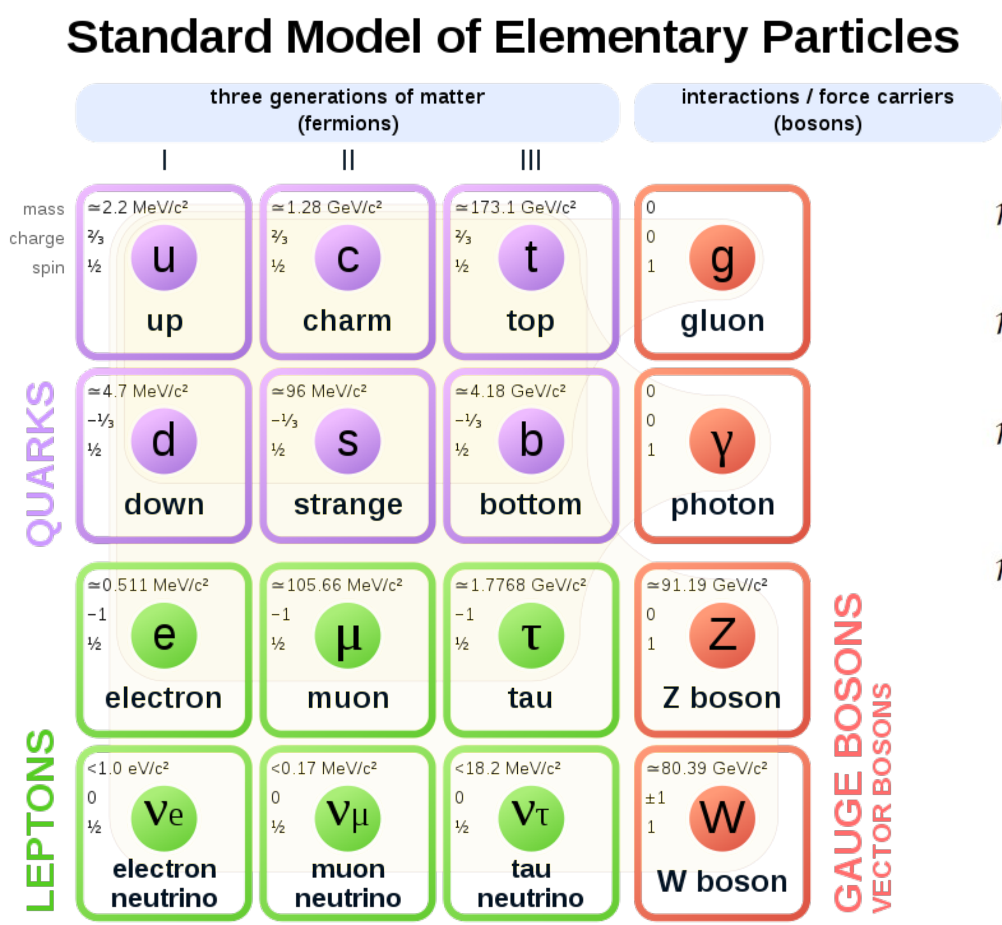
??

But: $m_{\rm proton} \neq m_{\rm u\,quark} + m_{\rm u\,quark} + m_{\rm d\,quark}$

$$m_{\rm u\,quark} \approx m_{\rm d\,quark} \approx m_{\rm electrons} \sim \frac{1}{1000}\, m_{\rm proton}$$

$$\boxed{m_{\rm proton} >> m_{\rm u\,quark} + m_{\rm u\,quark} + m_{\rm d\,quark}}$$





$$m_{\text{c quark}} \approx m_{\text{proton}}$$

$$m_{\text{b quark}} \approx 4\, m_{\text{proton}}$$

$$m_{\text{t quark}} \approx 180\, m_{\text{proton}}$$

$$m_{\tau\,\text{lepton}} \approx 4000\, m_{\text{electrons}} \approx 2\, m_{\text{proton}}$$

Mass of nucleons are larger than their composite quarks. Even though nucleus mass is less than its composite nucleons.

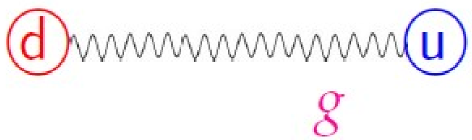
Are heavy quarks and leptons composite particles? No, there is no evidence whatsoever for their internal structure.

How do we then explain their large masses?

Answer: the Higgs mechanism

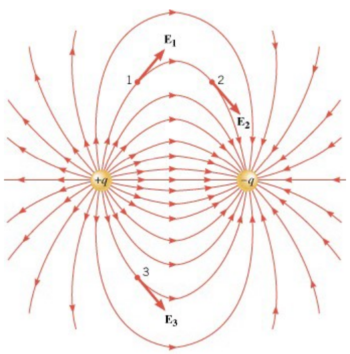
Interaction

Gluon is the exchange boson of the strong interaction between quarks

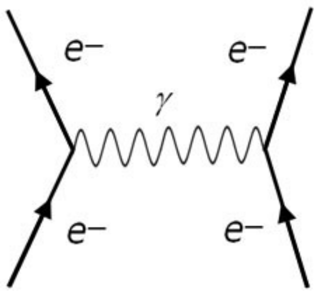


Bosons ($W^{\pm}$, W^0, Z) carry the weak interaction

Classical explanation: A charged particle creates a field in the surrounding space.

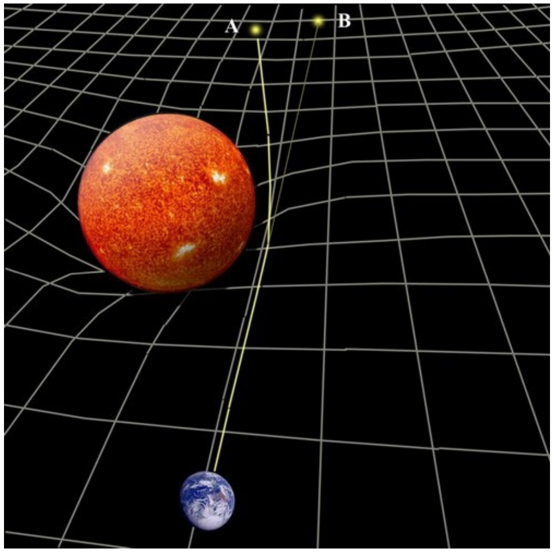


Quantum explanation (Feynman): a charged particle emits a photon (a boson, in general) and a second particle absorbs it.



Relativity: light bending

The photon (light) has nonzero mass due to its energy; it is deflected in the gravitational field, e.g. of the Sun



Some particles, e.g. the photon (light), the neutrino ν. . . *travel with the* speed of light c.

$$m_0 = m \times \sqrt{1 - \frac{v^2}{c^2}} = 0, \quad \text{if} \quad v = c$$

Particles that travel with the speed of light have zero rest mass.
Vice versa: Massless particles cannot rest; they always travel with the speed of light.
Strange behaviour? Not at all; according to Higgs, there is nothing wrong with massless particles; what is strange is that the "normal“ bodies rest or travel with the speed less than the speed of light.

Higgs field

Higgs assumed the existence of a new field, – the Higgs field, – that fills all of space and has no external source. The Higgs boson is an elementary excitation of the field.

The source of the Higgs field is the Higgs field itself. In the alternative picture, the Higgs bosons in the condensate attract each other. The resulting potential energy of the system has its minimum at a non-zero value of the field.

The Vacuum consists of many fields and particles are elementary excitations of the fields.

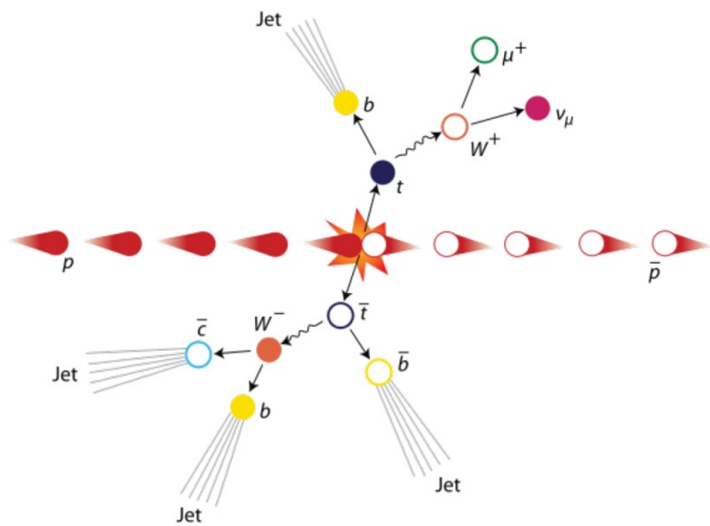
Mass generation

All elementary particles are massless and therefore move with the speed of light. But most of them bounce off the Higgs bosons in the vacuum and hence effectively move with a finite velocity. Their kinetic energy is transformed into the rest energy (mass).

Some particles – including the Higgs boson itself – interact more frequently than the others; it means they are more massive. Photons, gluons, neutrinos do not interact at all; they are massless – more precisely, their rest energy is zero.

Discovery of new boson

At the collider two proton traveling in opposite direction collide and produce a shower of particles, mostly quark-antiquark pairs, which in turn annihilates and produce long lived particle that are finally detected and analyzed.



The particle decays into at least some of the predicted channels by the Standard Model .

Moreover, the production rates and branching ratios for the observed channels match the predictions by the Standard Model within the experimental uncertainties.

So far the observations are consistent with the observed particle being the Standard Model Higgs boson.

Symmetry in Physics: What is Symmetry?

- A symmetry operation is a mathematical operation which leaves the final state indistinguishable from the initial state. For example: the sphere is considered to be the most perfectly symmetric geometric figure because any rotation about any axis or any reflection through any plane will leave the sphere indistinguishable from its original state. A cylinder is less symmetric because it has only one axis about which any rotation will leave it unchanged.
- A system is said to possess a symmetry if one can make a change in the system such that, after the change, the system appears exactly the same as before.
- Symmetry is an invariance of an object or system to a set of changes (transformations).

For examples

- Any fundamental experiment that we do in our laboratory will have exactly the same result if we move our laboratory to different place, country or planets. In word of symmetry, the laws of physics are independent of where we are in the universe. (symmetry under transformation of the space coordinates)
- If we look at light emitted by a distant star a billion years ago we observe that its wavelength is exactly the same as would be produced in the same atomic process today. In symmetry words, the laws of physics are independent of the passage of time. (symmetry under transformation of the time coordinate.
- If we do experiment on earth or on a flying satellite will give the same results as those done in our lab show that the laws of physics are independent of motion
- In symmetry language, the workings of the universe are symmetric with respect to position, orientation, time, and velocity.

Symmetry and Maxwell's Equations

$$\oiint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0} \qquad \oiint \vec{B}\cdot d\vec{A} = 0 \qquad \oint \vec{E}\cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B}\cdot d\vec{s} = \mu_0 i$$

- Symmetry was invoked to say that if an electrical current can produce a magnetic field then a magnetic field should be able to produce an electrical current.
- Maxwell again recognized that if the laws of physics were to be orderly and symmetric, the last of the above equations needed another term: if a changing magnetic field induces an electric field then a changing electric field should induce a magnetic field! Using arguments that vector fields must be continuous, Maxwell added a term to the last equation to make it:

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

- There is still one possible symmetry missing from Maxwell's Equations
- If there were such a thing as a magnetic charge, m, also called a *magnetic monopole, and we represent a current of these charges as "p," then the above* equations would be perfectly symmetric with respect to interchanging electric and magnetic charges and currents

$$\oiint \vec{E}\cdot d\vec{A} = \frac{q}{\varepsilon_0} \qquad \oint \vec{E}\cdot d\vec{s} = \frac{p}{\varepsilon_0} - \frac{d\Phi_B}{dt}$$

$$\oiint \vec{B}\cdot d\vec{A} = \mu_0 m \qquad \oint \vec{B}\cdot d\vec{s} = \mu_0 i + \mu_0\varepsilon_0 \frac{d\Phi_E}{dt}$$

Maxwell's equation can be written as

$$\nabla \cdot \vec{D} = \rho \qquad \nabla \cdot \vec{H} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial \tau} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial \tau} + \vec{j}$$

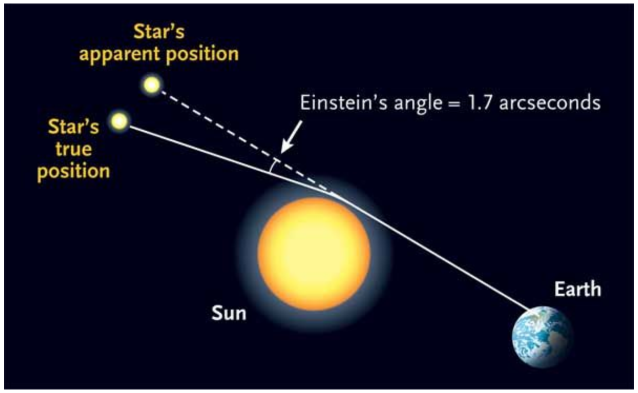
(where ρ is charge density, $\vec{j}$ is vector current density, $\vec{D} = \varepsilon_0 \vec{E}$ and $\vec{B} = \mu_0 \vec{H}$)

Maxwell's equation can be written for magnetic charge of density φ and vector current $\vec{z}$

$$\nabla \cdot \vec{D} = \rho \qquad \nabla \cdot \vec{H} = \varphi \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial \tau} + \vec{z} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial \tau} + \vec{j}$$

Symmetry and Einstein's Special and General Relativity

- Special Relativity and General Relativity formulated by Einstein are fundamentally extensions of symmetry ideas.
- In Special Relativity, Einstein simply postulated that the invariance of physical laws with respect to motion at constant velocity
- Famous results of length contraction and time dilation, lead the fact that space and time are intermingled, that seem so revolutionary in Special Relativity.
- General Relativity is built on a more subtle observed symmetry of our universe: the fact that no experiment can distinguish between the effects of a gravitational force and the effects of an accelerated frame of reference.
- The leads immediately to the prediction that light must "fall" in a gravitational field just as a stone does.



Symmetry and Modern Physics

- Eugene Wigner's introduction of symmetry groups into Quantum Mechanics, moved to the forefront of nearly all thinking in modern physics.
- One of the most striking examples of the use of symmetry ideas in the advancement of modern physics is in the development of what is know known as "The Standard Model" of elementary particles.
- By the 1930's, our knowledge of the fundamental structure of matter, all atoms were composed of just three different entities: protons, neutrons and electrons.
- By the late 1950‘s, physicists managed to produce more than thirty new "elementary particles," with no end in sight - always the kind of thing that makes physicists think "the universe must be much simpler and more elegant than that.
- Then, in 1963, Murray Gell-Mann discovered that all of the known particles governed by the strong nuclear force could be placed into a particular mathematical symmetry group called SU(3). Each of these "elementary particles" has a definite mass, electric charge, and a set of characteristics (quantum numbers) named, for historic reasons, "spin," "isospin," "hypercharge," and "baryon number.

Gauge Symmetry

- The mechanism by which it did so (the Higgs mechanism) involves a quantum field (the Higgs field), which has a non-zero value associated with every point in space. The Higgs particle is a ripple, a parcel of energy, in the Higgs field.
- The Higgs field tugs on W and Z particles, restricting their communication of the weak force to an extremely short range (less than about one-ten-thousand-trillionth of a centimetre). In other words, it gives the W and Z particles inertia, or mass. In similar fashion, the molasses-like Higgs field gives mass to other fundamental particles, such as electrons and quarks.
- Because the vacuum does not carry electrical charge, the photon travels unhindered. So the photon remains massless and can render the electromagnetic force over long distances.

Higgs mechanism and electroweak unification

The Higgs mechanism plays a key role in the physics of elementary particles: in the context of the Standard Model, the theory which, describes in a unified framework the electromagnetic, weak, and strong nuclear interactions, it allows for the generation of particle masses while preserving the fundamental symmetries of the theory. This mechanism predicts the existence of a new type of particle, the scalar Higgs boson, with unique characteristics. The detection of this particle and the study of its fundamental properties is a major goal of high-energy particle colliders, such as the CERN Large Hadron Collider or LHC.

Supersymmetry

One of the major steps beyond the standard model involves supersymmetry — the idea that each particle we know has a not-yet-discovered superpartner.